

AD-A037 973

BROWN UNIV PROVIDENCE R I DIV OF ENGINEERING
CONTROL OF NON-LINEAR SYSTEMS. (U)
DEC 76 A E PEARSON

F/G 12/1

UNCLASSIFIED

AFOSR-TR-77-0285

AF-AFOSR-2793-75
NL

| OF |
AD
A037973



END

DATE
FILMED
4-77

AFOSR - TR - 77 - 0285

Progress Report

for

CONTROL OF NONLINEAR SYSTEMS

to

Directorate of Mathematical and Information Sciences
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
AFOSR/NM, Building 410
Bolling Air Force Base
Washington, DC 20332

Grant: AFOSR-75-2793

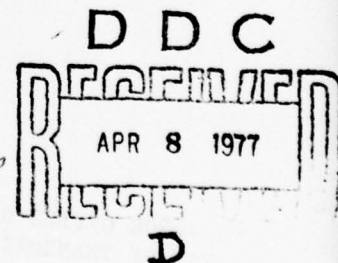
Period Covered: January 1, 1975-December 31, 1976

Brown University
Division of Engineering
Providence, Rhode Island 02912

PRINCIPAL INVESTIGATOR: Allen E. Pearson

December 31, 1976

Approved for public release;
distribution unlimited.



AD NO. _____
DDC FILE COPY

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

17-07-4207A

350530AQA

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

DDC LIFE CYCLE
VO 110

19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER	
AFOSR - TR - 77 - 0285			
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED	
CONTROL OF NON-LINEAR SYSTEMS		Interim	
		6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s)	
Allan E. Pearson		15 VAF - AFOSR - 2793 - 75	
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
Brown University Division of Engineering Providence, Rhode Island 02912		61102F 17 2304/A1	
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE	
Air Force Office of Scientific Research/NM Bolling AFB DC 20332		Dec 31 1976	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES	
12/17 p.		17	
		15. SECURITY CLASS. (of this report)	
		UNCLASSIFIED	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)			
Approved for public release; distribution unlimited			
9 Progress rept. 1 Jan 75 - 31 Dec 76			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)			
Linear Differential System, Sufficient Conditions, Parameter Identification			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)			
New methods were developed for stabilizing a "time-varying" linear differential system. Three technical notes involving feedback stabilization of a discrete-time constant linear system, derivation new lower bounds on the solution matrix to the algebraic matrix Riccati equation, and new sufficient conditions for the linear constant differential-difference system were published. Results have been obtained concerning a minimum energy regulator problem for linear time-invariant systems in which the control variable is subject to an "average-power" constraint on the response time interval.			

TABLE OF CONTENTS

	PAGE
I. INTRODUCTION	1
II. SUPPORTED PERSONNEL	1
III. RESEARCH COMPLETED	2
a. Control of Linear Systems	2
b. Control of Nonlinear Systems	7
c. Parameter Identification	10
IV. REFERENCED PUBLICATIONS	14

ACCESSION BY	
NTIS	White Section <input checked="" type="checkbox"/>
DEC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
A	

DDC
RECEIVED
 APR 8 1977
RECEIVED
 D

I. INTRODUCTION

This progress report concerns research completed to date under grant AFOSR-75-2793. The personnel listed in Section *II* received at least partial support from the grant during this period. Completed research is discussed in Section *III*. Publications, including papers submitted for publication, are listed in Section *IV*.

II. SUPPORTED PERSONNEL

Y. K. Chin, Research Assistant

W. H. Kwon, Research Associate

J. M. Mocenigo, Research Assistant

A. E. Pearson, Professor of Engineering

K. C. Wei, Research Assistant

III. RESEARCH COMPLETED

(a) Control of Linear Systems

Although the synthesis of feedback control laws for linear differential systems has been actively researched for many years, there exist very few methods for stabilizing a "time-varying" linear differential system. New results on this class of problems were obtained by Kwon and Pearson, [1-3], relative to the linear time varying system

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0 \\ y(t) &= C(t)x(t)\end{aligned}$$

where the matrices $(A(t), B(t), C(t))$ are assumed to be piecewise continuous functions for all $t \geq t_0$. The feedback control law synthesized for this system is of the form

$$u(t, x) = -R^{-1}(t)B'(t)P^{-1}(t, t+T)x$$

where the nonsingular symmetric matrix $P(t, t+T)$ is obtained by integrating the matrix Riccati equation

$$\begin{aligned}\frac{-\partial P(\tau, \sigma)}{\partial \tau} &= -A(\tau)P(\tau, \sigma) - P(\tau, \sigma)A'(\tau) \\ &\quad - P(\tau, \sigma)C'(\tau)Q(\tau)C(\tau)P(\tau, \sigma) \\ &\quad + B(\tau)R^{-1}(\tau)B'(\tau), \quad \tau \leq \sigma\end{aligned}$$

backward in time from $\tau = \sigma = t+T$ to $\tau = t$, subject to the boundary condition $P(\sigma, \sigma) = 0$. It is shown in [3] that the above feedback control law is optimal for the moving cost function

$$J(u) = \int_t^{t+T} [y'(\tau)Q(\tau)y(\tau) + u'(\tau)R(\tau)u(\tau)]d\tau$$

subject to the moving terminal constraint $x(t+T) = 0$, where $(Q(\cdot), R(\cdot))$ are

nonnegative definite symmetric weighting matrices with $R(t) > 0$ for all t , and T is a chosen positive scalar. More importantly, the above control law has been shown to be uniformly asymptotically stable under some mild technical conditions involving controllability and observability of the matrix pairs $(A(t), B(t))$ and $(A(t), C(t))$ and the choice in the parameter $T > 0$. A major advantage of the above control law in comparison with the standard regulator problem is that the integration interval is finite for the Riccati equation of this formulation, while it is infinite for the solution to the standard regulator problem. Also, it is shown in [3] that the minimal values of the cost functions for the above receding horizon problem and the standard regulator problem are identical for $T = \infty$, thus providing a link between the two types of linear state variable feedback control law solutions.

In the case of time invariant systems with constant weighting matrices, i.e., (A, B, C, Q, R) all constant, the above control becomes a fixed gain feedback control law and, as shown in [3], generalizes a well-known method for stabilizing a linear fixed system given by Kleinman^{*}. In particular, Kleinman's result is obtained as a special case by choosing $Q = 0$ for the weighting matrix on the state.

The dual problem in filtering theory corresponding to the above control problem is shown in [3] to yield an asymptotically stable Kalman-Bucy filter for the case of completely unknown statistics involving the initial state $x(t_0) = x_0$, i.e., the case in which the mean $E\{x_0\} = \bar{x}_0 = \text{unknown}$ and the variance $E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)'\} = \infty$.

Three technical notes pertaining to the control of linear systems have also been completed during this period. The first, [4], extends a result due

^{*}Kleinman, D. L., "An Easy Way to Stabilize A Linear Constant System," IEEE Trans. Auto. Contr., AC-15, 692, December, 1970.

to Kleinman^{*} in the feedback stabilization of a discrete-time constant linear system, $x(i+1) = Ax(i) + Bu(i)$, by a feedback control of the form

$$u(i) = -R^{-1}B'A^N[\epsilon I + \sum_{k=m}^N A^k B R^{-1} B'A^k]^{-1} A^{N+1} x(i)$$

where the choice in the integers (m, N) and the nonnegative scalar ϵ depend on the multiplicities of the zero eigenvalue as a root of the characteristic and minimal polynomials of the matrix A . The main result of this note is the removal of the nonsingularity condition on the A matrix and a weakening of the controllability assumptions pertaining to the pair (A, B) .

The second technical note, [5], derives new lower bounds on the solution matrix K to the algebraic matrix Riccati equation, $A'K + KA - KBB'K + Q = 0$, and shows how these bounds are sharper than those appearing previously in the literature, as well as providing exact estimates in certain special cases. Extensions to the discrete algebraic matrix Riccati equation are also included in [5].

A third technical note, [6], provides new sufficient conditions for the linear constant differential-difference system

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + Bu(t)$$

to be memoryless stabilizable by a feedback control law of the form

$$u(t) = F x(t).$$

More importantly, the results in [6] are constructive in that the gain matrix F can be easily computed if it is determined that the various derived sets of sufficient conditions for stabilizability are upheld. One such set of sufficient conditions is the existence of a positive definite matrix Q and a

^{*}Kleinman, D. L., "Stabilizing a Discrete, Constant, Linear System With Application to Iterative Methods for Solving the Riccati Equation," IEEE Trans. on Auto. Contr., AC-19, pp. 252-254, June 1974.

positive scalar T such that the matrix inequality

$$4A_h' Q^{-1} A_h' < 2BB' + P(T)QP(T)$$

is upheld, where $P(T)$ is the solution (at a fixed time T) of the matrix Riccati differential equation

$$\dot{P}(t) = -AP(t) - P(t)A' - P(t)QP(t) + BB', \quad P(0) = 0.$$

In this case, the stabilizing gain matrix F is given by $F = -B'P^{-1}(T)$. Another (simpler) set of sufficient conditions is given by the selection of the matrix Q according to the inequality

$$Q \geq 2\lambda_{\max}(HH')I$$

where H is a matrix such that $A_h = BH$. This case applies to that special situation in which the columns of A_h are linear combinations of the columns of B . The gain matrix F is defined the same as in the first case after defining Q so as to satisfy the above inequality. This special case is not devoid of representation since it is shown by way of example in [6] that such a Q can be constructed for the linear constant differential difference system

$$\sum_{i=0}^n \alpha_i y^{(i)}(t) + \sum_{i=0}^n \beta_i y^{(i)}(t-h) = u(t)$$

where $y^{(i)} \triangleq i^{\text{th}}$ derivative of y .

Results have been obtained during this period concerning a minimum energy regulator problem for linear time-invariant systems in which the control variable is subject to an "average-power" constraint on the response time interval [7]. The optimization problem considered is that of minimizing the control energy cost function,

$$J(u) = \int_{t_0}^T u'(t)u(t)dt$$

for the linear system $\dot{x} = Ax + Bu$, while regulating the state of the system from

the given initial state $x(t_0) = x_0$ to the origin $x(T) = 0$ in a fixed time $T - t_0$. After solving this problem (the solution of which is well-known), the response time $(T - t_0)$ is chosen so that the optimal control signal \hat{u} satisfies the average-power constraint

$$\frac{1}{T - t_0} \int_{t_0}^T \hat{u}(t) \hat{u}(t) dt = 1.$$

This is an optimal controller with transient response characteristics comparable to a time optimal "bang-bang" controller in which each control signal is subject to the hard constraint $|u_i(t)| \leq 1$ on the response time interval $t_0 \leq t \leq T^*$, and T^* is the minimum time solution to reach the origin. The above average-power constraint is also satisfied (coincidentally) by the bang-bang controller, which accounts for the similarity in their transient response characteristics. However the minimum energy average-power constrained regulator is easier to obtain in feedback form due to the softer constraint on the control variable. Although also implicitly defined, the main advantage of this control law over the time optimal control is that a suboptimal, explicitly defined, feedback control can be constructed, as shown in [7], with whatever degree of accuracy is desired for a general n^{th} order system, while this is practically impossible for the time optimal bang-bang controller when $n \geq 3$. It is noted that the control law for the minimum energy average power constrained regulator is nonlinear, and that the asymptotic stabilization of this nonlinear control law has been established in [7]. Simulation results for second and fourth order examples are also summarized in [7] which illustrate the restraining effect of the average power constraint on the control signal while regulating the state of the system over a wide range of initial conditions in the state space. This is the major advantage of this nonlinear controller over a linear feedback controller designed to achieve the same settling time.

(b) Control of Nonlinear Systems

Sufficient conditions for the controllability of the class of nonlinear systems described by

$$\dot{x}(t) = A(t, x(t), u(t))x(t) + B(t)u(t) + f(t, x(t), u(t)), t_0 \leq t \leq t_1$$

have been obtained by Wei [8] during this period. These conditions involve the nonsingularity of the controllability Gramian associated with the parametrized matrix pair $\{A(t, \zeta(t), v(t)), B(t)\}$, where $\zeta(t)$ and $v(t)$ are regarded as elements (parameters) in a product space, $C_{nm}[t_0, t_1]$, of vector valued continuous function pairs, $(\zeta(t), v(t))$, on the time interval $t_0 \leq t \leq t_1$. Using the Schauder's fixed point theorem in $C_{nm}[t_0, t_1]$, sufficient conditions for both local and global controllability are derived involving the boundedness and continuity of the quantities $(A(t, x, u), B(t), f(t, x, u))$ and their partial derivatives, in addition to the nonsingularity of the aforementioned controllability Gramian. These results remove some assumptions previously needed in earlier publications on this problem and, generally, extend these earlier results to a broader class of nonlinear systems.

A number of results have been obtained during this period concerning the bilinear regulator problem for the class of nonlinear systems described by

$$\dot{x}(t) = (A + \sum_{i=1}^m B_i u_i(t))x(t)$$

where $u = (u_1, \dots, u_m)'$ is the control variable and (A, B_1, \dots, B_m) are given $n \times n$ matrices, [9-10]. First, existence of an optimal control has been established for the minimization of the quadratic cost

$$J(u) = x'(T)Qx(T) + \int_{t_0}^T [x'(t)W(t)x(t) + u'(t)R(t)u(t)]dt$$

where $(Q, W(t), R(t))$ are symmetric nonnegative definite weighting function matrices

with $R(t) > 0$ for all $t \in [t_0, T]$. Next, the regulator problem for the special class of commutative bilinear systems has been considered in some detail. This is the class for which every pair of matrices in the set $\{A, B_1, \dots, B_m\}$ commute with each other. Within the context of this class, it has been shown that the optimal control which minimizes the above quadratic cost, without any terminal constraint on the state, is in the form of a constant vector which satisfies a certain nonlinear algebraic equation. Furthermore, for a single input commutative bilinear system ($m=1$), it is shown in [10] that this optimal control is unique if (as a sufficient condition) the matrix $B_1'Q + B_1'QB_1$ is non-negative definite. Also, sufficient conditions have been obtained in the multi-input case which involve the nonnegative definiteness for all $v \in \mathbb{R}^n$ of the $m \times m$ matrix $Z(v)$ defined by

$$Z_{ij} = v'(B_i' B_j' Q + B_i' Q B_j) v, \quad i, j = 1, \dots, m.$$

The implication of these results for the regulator problem associated with a commutative bilinear system is that the optimal control can be computed by well-known iterative methods in finite dimensional (\mathbb{R}^m) spaces, and that this control vector is unique if certain additional conditions involving the system matrices are upheld.

Concerning the same class of regulator problems for commutative bilinear systems, but with a fixed terminal state constraint, i.e., $x(T) = x_1 = a$ given terminal vector, it has also been shown in [10] that if x_1 belongs to the reachable set, then there exists a constant optimal control which does the job, and that this optimal control vector satisfies a certain nonlinear algebraic equation which depends on the given boundary conditions: $x(t_0) = x_0$ and $x(T) = x_1$. In the terminal constraint problem such optimal controls are not generally unique and a simple example is given in [10] to illustrate this fact.

Application of the above theory for the regulator problem of a commutative bilinear system has been obtained in the companion paper [11]. Here it is shown how the two dimensional missile intercept problem for a maneuverable target and a pursuing missile can be formulated in the present context through the introduction of some auxiliary states. The kinematic equations are

$$\begin{aligned}\dot{x}_1(t) &= -v_T \sin x_3(t) + x_2(t)u_p(t) \\ \dot{x}_2(t) &= v_T \cos x_3(t) - x_1(t)u_p(t) - v_p(t) \\ \dot{x}_3(t) &= u_T - u_p(t)\end{aligned}$$

where (v_T, v_p) are the line speeds of the target and pursuer, (x_1, x_2) are the position coordinates of the target relative to the pursuer, x_3 is the relative angle between the headings of the two missiles, and (u_T, u_p) are the angular rates of the target and pursuer. Introducing auxiliary states $x_4 = \sin x_3$, $x_5 = \cos x_3$ and $x_6 = 1$, and making the crucial assumption that the line speed v_p of the pursuer can be modeled proportional to u_p , i.e., $v_p(t) = \gamma u_p(t)$, it is first shown in [11] that the resulting equations of motion are in the bilinear form, $\dot{x} = (A + Bu_p)x$, and that the 6x6 matrices (A, B) commute. This implies that the optimal control for the quadratic cost problem is a constant vector satisfying a certain nonlinear algebraic equation. Furthermore, it is possible to solve these nonlinear equations explicitly, i.e., in closed-form, for the terminal constraint case: $x_1(T) = x_2(T) = 0$, i.e., zero missed distance, given that the intercept angle $\beta = x_3(T)$ is allowed to be selected with some degree of latitude. Specifically, it has been shown that there exists a triple (γ, β, T) for every set of initial data $(x_1(t_0), x_2(t_0), x_3(t_0))$ such that the desired zero missed distance terminal constraint can be upheld, and that the optimal control u_p^* which minimizes the quadratic cost

$$J(u) = \int_{t_0}^T u^2(t) dt$$

subject to $x_1(T) = x_2(T) = 0$, is a constant given by

$$u_p^* = u_T + \frac{x_3(t_0) - \beta}{T - t_0}$$

Inasmuch as this solution has been obtained in closed-form, it is potentially feasible that the result might be used on-line for obtaining a closed-loop control law for the missile intercept problem assuming that the target speed, acceleration and initial heading, $(v_T, u_T, x_3(t_0))$, can be estimated from the given measurements. A least squares estimate of the pair of quantities $(v_T, x_3(t_0))$ has been derived, and the entire step-by-step estimation and control sequence, which defines the closed-loop control law, has been simulated under a variety of initial conditions. A summary of these simulation runs is given in [11].

A singular perturbation problem has also been considered in [11] relating to the practical situation in which the missile turn rate is furnished by a motor with actuator dynamics. First order dynamics were assumed for the analysis and simulation studies, but the results actually apply to higher order actuator dynamics as well. An interesting feature of these results is that a closed-form solution can be obtained for the higher order singularly perturbed system of this paper in contrast with the approximate solutions for general nonlinear systems.

(c) Parameter Identification

A deterministic least squares identification of the coefficient matrices in the differential operator model

$$P(D)y(t) = Q(D)u(t), \quad D = \frac{d}{dt}$$

where

$$P(D) = D^n + \sum_{i=0}^{n-1} P_{n-i} D^i, \quad Q(D) = \sum_{i=0}^{n-1} Q_{n-i} D^i,$$

has been developed in [12,13] which differs from more traditional uses of least squares theory in the following respects: (i) input-output data $[u(t), y(t)]$ is assumed to be given on a finite time interval, $0 \leq t \leq t_1$, of arbitrarily short (but non-zero) duration, (ii) unknown disturbance inputs and measurement noises on $0 \leq t \leq t_1$, are modeled implicitly in the above model by arbitrary solutions to a homogeneous linear differential equation of assumed order, but with no assumptions about the characteristic modes of this equation, (iii) no attempt is made to estimate either the initial state of the system or the initial conditions giving rise to the disturbance inputs on $0 \leq t \leq t_1$.

One advantage of this approach, which might be termed parameter identification without initial state estimation, is that the potential exists for obtaining very accurate estimates of the system parameters, based on input-output data observed over a relatively short time interval, even for very small signal-to-noise ratios, eg. -20db. or less. The main reason for this lies in the technique developed in [12,13] for circumventing the need to estimate the unknown initial conditions, which reduces this aspect of the computational burden associated with other approaches. Another reason is that the disturbances are modeled deterministically as uncontrollable modes, and the frequencies associated with these modes on $0 \leq t \leq t_1$ are identified along with the system parameters.

Theoretical conditions for the uniqueness of solutions to the above least squares estimation problem have also been obtained in [13]. These conditions involve the linear independence of the given input-output data, together with a certain number of their derivatives on $[0, t_1]$. Simulation results are

reported in [13] which illustrate that highly accurate estimates for the parameters of a fourth order system can be obtained on a time interval comparable to the time constants in the system even in the presence of very large disturbance signals.

Subsequent to the results reported in [12,13], important extensions have been obtained which enlarge the class of systems and provide for computational advantages in a variety of situations [14]. These extensions arise principally by viewing the identification problem in terms of finding a parameter vector θ which satisfies a differential operator equation of the form

$$P(D)v(t) + Q(D)V(t)f(\theta) = 0, \quad 0 \leq t \leq t_1$$

where $(P(D), Q(D))$ are given polynomial matrices in the differential operator $D = \frac{d}{dt}$, $(v(t), V(t))$ are vector and matrix valued functions of the given input-output data, $f(\theta)$ is a given vector valued function (possibly nonlinear) of the parameter vector θ , and the observation time interval, $0 \leq t \leq t_1$, is again of arbitrarily short duration. Some attributes of this formulation in relation to the results reported in [12,13] are the following: (i) the parameters θ may enter nonlinearly into the basic model, i.e., the function $f(\cdot)$ may be a nonlinear function of θ , (ii) the disturbances on $0 \leq t \leq t_1$ are modeled exactly the same as in [12,13], i.e., by arbitrary solutions to a homogeneous differential equation of assumed order, but with no assumptions about the characteristic modes of this equation; however, the parameters associated with the disturbances are modeled explicitly in the formulation in [14], in contrast with the implicit modeling of disturbances in [12,13], (iii) the coefficient matrix of the highest derivative on the input-output data is allowed to be singular for the formulation in [14], while this condition was previously ruled out due to the particular state variable representation used in [12,13].

The computational aspects of the formulation in [14] involve minimizing an explicitly defined function $J(\theta)$ of the form

$$J(\theta) = f'(\theta)\Phi f(\theta) + 2\zeta'f(\theta) + \alpha$$

where the nonnegative definite matrix Φ , the vector ζ and the scalar α are determined by integrating a certain set of differential equations driven by the input-output data on $0 \leq t \leq t_1$. Moreover, this minimum is known to correspond to the sought value of $\theta = \theta^*$ if $J(\theta^*) = 0$ and some other nondegeneracy conditions are upheld involving the input-output data (see the Assertion on p. 846 of [14]).

REFERENCED PUBLICATIONS

- [1] Kwon, W. H. "Infinite-Time Regulator for a Class of Functional Differential Systems and a Minimum Control Energy Problem for Ordinary Differential Systems," Ph.D. Thesis, Brown University, June 1976.
- [2] Kwon, W. H. and Pearson, A. E., "A Modified Minimum Energy Regulator Problem and Feedback Stabilization of a Linear System," Proc. of 1976 Conf. on Inform. Sci. and Syst., Johns Hopkins Univ., pp. 129-134, April 1976.
- [3] Kwon, W. H. and Pearson, A. E., "A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System," (Full length paper submitted 11/29/76 to the IEEE Trans. on Auto. Contr.)
- [4] Kwon, W. H. and Pearson, A. E., "On the Stabilization of a Discrete Constant Linear System," IEEE Trans. on Auto. Contr., Vol. AC-20, pp. 800-801, December 1975.
- [5] Kwon, W. H. and Pearson, A. E., "A Note on the Algebraic Matrix Riccati Equation," To appear in IEEE Trans. on Auto. Contr., Vol. AC-22, No. 1, February 1977.
- [6] Kwon, W. H. and Pearson, A. E., "A Note on Feedback Stabilization of a Differential-Difference System," (Submitted 8/11/76 to IEEE Trans. on Auto. Contr.)
- [7] Pearson, A. E. and Kwon, W. H., "A Minimum Energy Feedback Regulator for Linear Systems Subject to an Average Power Constraint," IEEE Trans. on Auto. Contr., Vol. AC-21, pp. 757-761, October 1976.
- [8] Wei, K. C., "A Class of Controllable Nonlinear Systems," IEEE Trans. on Auto. Contr., Vol. AC-21, pp. 787-789, October 1976.
- [9] Wei, K. C., "Optimal Control of Bilinear Systems With Some Aerospace Applications," Ph.D. Thesis, Brown University, June 1976.
- [10] Wei, K. C. and Pearson, A. E., "On the Bilinear Regulator Problem With a Pursuit-Evasion Application," (Submitted 6/22/76 to the IEEE Trans. on Auto. Contr.)
- [11] Wei, K. C. and Pearson, A. E., "Minimum Energy Control of a Bilinear Pursuit Evasion System," (Submitted 9/9/76 to the AIAA Journal.)
- [12] Pearson, A. E., "Identification of Linear Differential Systems from Input-Output Without Estimating the Initial State," Proc. of 1975 Conf. on Inform. Sci. and Syst., pp. 387-390, Johns Hopkins University, April 1975.
- [13] Pearson, A. E. "Finite Time Interval Linear System Identification Without Initial State Estimation," To appear in Automatica, November 1976.
- [14] Pearson, A. E., "Positive Definite Performance Functions for Parameter Adaptive Control Problems," Proc. of 1975 IEEE Conf. on Decis. and Contr., pp. 844-849, Houston, TX, December 1975.

OTHER PUBLICATIONS DURING THIS PERIOD

- [15] Koopersmith, R. M. and Pearson, A. E., "Determination of Trajectories for a Gliding Parachute System," U.S. Army Natick Labs Report 75-117-AMEL, April 1975.
- [16] Pearson, A. E. and Wei, K. C., "Control of a Gliding Parachute System in a Nonuniform Wind," U.S. Army Natick Labs Report TR-76-60-AMEL, May 1976.

REPORT PREPARED BY: Allan E. Pearson

FOR GRANT: AFOSR-75-2793

Allan E. Pearson

Allan E. Pearson
Professor of Engineering
Principal Investigator

Carl Cometta

Carl Cometta
Executive Officer
Division of Engineering